

ESTIMATION OF THE THERMAL AND EROSION ACTION  
OF A SUPERSONIC DUSTY STREAM ON A BLUNT CONE

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UDC 533.6.011

The influence of the interaction between phases on the thermal and erosive action of dust particles on the surface of a blunt cone over which a supersonic two-phase stream flows is estimated on the basis of a model of two-velocity and two-temperature continuous medium, using empirical equations.

In the flow of a supersonic two-phase stream over a body, its surface is subjected to thermochemical and erosive action. Experimental studies of such flow have shown that in this case the heat fluxes in the vicinity of the critical point of a body are increased in comparison with the flow of "pure air" over the body. This is influenced by several factors, chief of which are the following. The reflection of coarse-fraction particles from the surface results in a considerable change in the structure of the stream in the vicinity of the critical point of a blunt body, as shown in [1-3]. The reflected particles and the particles produced by surface erosion alter both the parameters in the flow field and the shock wave. The latter acquires a doomed shape typical of flow over pointed bodies. This results, in turn, in an increase in the convective heat flux to the surface.

Intensification of heat transfer from the high-temperature gas stream occurs in the shock layer due to disruption of the laminar flow structure in the wall layer by particles incident on the surface and knocked from it.

The next and final effect that significantly influences the heat flux and the erosive destruction of the surface is the direct action of dust particles incident on the surface.

The possibilities for experimental study of all of these effects are very limited. This is due to the difficulties in obtaining dusty supersonic streams with a uniform field of gas-dynamic parameters and particle parameters under laboratory conditions and to the complexities in measuring the required parameters near the surface [4, 5]. Experimental data on the thermal and erosive action of a dusty gas on a body are therefore limited to the basic case of standard bodies: a disk and a sphere.

A complete theoretical description of the processes of interaction of a dusty gas stream with the surface of a body is still lacking. There are models that enable one to study, to a certain extent, some relationships in the flow of two-phase media. One of the efficacious models is that of interpenetrating continua [6]. Most results in the solution of specific problems have been obtained within its framework. A more complete investigation makes it necessary to allow for particles reflected from the surface. The reflection of particles results in the appearance of some of the effects indicated above. Models of a three- and four-velocity continuous medium are used to calculate flow over a body with allowance for reflected particles [7]. A more general and universal approach to calculating two-phase flow over bodies with allowance for reflected particles can be formulated on the basis of the Lagrange-Euler description of a two-phase medium [8]. Those models enable one, in principle, to determine the parameters of the gas and particles behind the shock wave, but further development and refinement are needed for their extensive practical application.

The above facts mean that a complete mathematical simulation of all of the processes associated with the thermal and erosive action of a dusty gas on a body over which it flows is not possible at present. But it is possible, under certain conditions, to estimate the influence of some of the aforementioned factors. One of them is the influence of the interaction between phases on the parameters of the particles incident on the surface of the body

and their thermal and erosive action on the surface. That is the effect that we shall discuss here. Although the study of that one effect does not, in general, enable one to allow fully for the influence of the dustiness of the gas on the quantities being determined, it does enable one to supplement the existing theoretical concepts about the phenomenon itself. The results discussed here may have practical importance in individual cases (the presence of small particles, when reflection from the surface does not introduce large changes into the stream; the adhesion of particles to the surface, etc.).

We shall consider flow conditions under which the momentum of the reflected particles is fairly low, those particles do not significantly distort the flow field, and the incident particles have the main influence on the thermal and erosive action. This occurs when fairly small particles are present in the gas and in flow over elongated bodies. The influence of the particles is manifested both through the changes in the gas-dynamic parameters caused by the interphase kinetic and thermal action and through the mechanism of conversion of particle energy into thermal energy in the collision of particles with the surface. In [1] it was shown that 70% of the total particle energy is converted into thermal energy in the collision with the surface. As follows from [9], the intensity of erosive destruction is proportional to the kinetic energy of the particles. In this connection, the intensity of the thermal and erosive action of a dusty stream is determined essentially by the parameters of the gas and particles at the surface of the body.

Various approximate equations, obtained in the analysis of experimental data, are used to determine thermal and erosive action.

The thermal energy flux produced by particles incident on the surface of a body may be determined from the equation [10]

$$q_s = K_a G_s \left[ c_s (T_s - T_w) + \frac{V_s^2}{2} \right], \quad (1)$$

where  $G_s = \rho_s (\mathbf{v}_s \cdot \mathbf{n}_w)$  is the mass of the incident particles; the coefficient  $K_a$  determines the fraction of the total particle energy imparted to the surface of the body. For liquid particles, or if a melted film is formed on the surface, we may take  $K_a = 1$ . For the collision of solid particles with a surface,  $K_a = 0.7$  was obtained in [1].

To estimate erosive ablation, we use an empirical equation suggested in [10, 11]:

$$G_{er} = \frac{G_s}{H_{er}} \frac{V_s^2}{2} F(\alpha), \quad F(\alpha) = \begin{cases} \frac{4}{3} \left( \sin \alpha - \frac{1}{4} \right), & \alpha > 30^\circ, \\ \frac{4}{3} (\sin \alpha)^2, & \alpha \leq 30^\circ, \end{cases} \quad (2)$$

where  $\alpha$  is the angle of attack.

The thermal and erosive action of particles in the flow of a dusty gas over a body is thus determined essentially by the parameters of the particles at the surface.

In a simplified approach, the heat flux from the incident particles and the rate of erosive ablation may be determined from the parameters of the undisturbed motion of the particles. The parameters of the gas are then assumed to equal the parameters corresponding to flow by a pure gas. In general, however, both the parameters of the gas and those of the particles are determined by the interaction between phases in the disturbed stream behind the shock wave. If the volume fraction of the solid phase is neglected, then in the transition through the shock wave, the parameters of the gas change in accordance with the Rankine-Hugoniot conditions, while the kinematic and energy parameters of the particles remain unchanged. In the shock layer the particles are decelerated and heated, and the increase in gas pressure, velocity, and density is more intense than for a pure gas. As for the gas temperature, its value is determined by two opposing mechanisms: gas cooling due to heat exchange between phases and gas heating due to dissipation of the kinetic energy of particles. In general, this results in nonmonotonic variation of the gas temperature in the shock layer and in a nonmonotonic dependence of the gas temperature at the surface of the body on the size of the particles [12].

The model of a two-velocity and two-temperature continuous medium [6] can be used effectively to describe the processes of interaction between phases under the conditions of the absence of reflection of particles from the surface of the body. In that model the par-

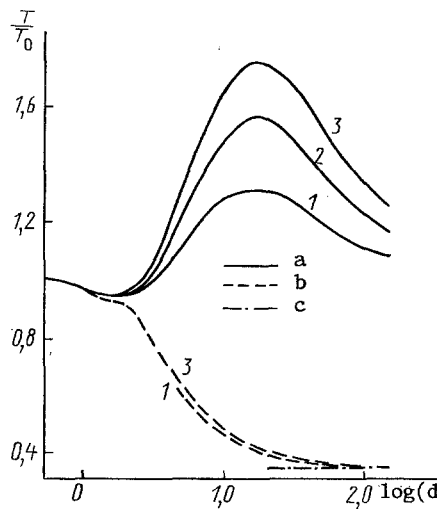


Fig. 1

Fig. 1. Gas and particle temperatures at the stagnation point on a sphere: a) gas temperature; b) particle temperature; c) temperature of gas and particles in the oncoming stream; 1)  $\rho_{S\infty} = 0.4$ ; 2) 0.8; 3) 1.2.

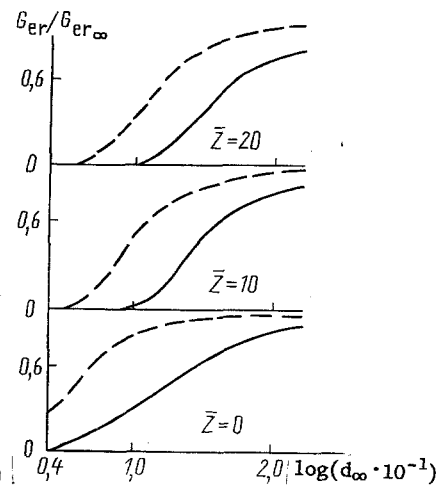


Fig. 2

Fig. 2. Linear rate of erosive ablation as a function of particle size.

ticle motion is described by the equations of a continuous medium devoid of intrinsic pressure. The interaction between gas and particles is taken into account by the inclusion of distributed mass forces, reflecting the exchange of momentum and energy.

The mathematical formulation of the problem and the expressions for the terms allowing for the exchange of momentum and energy between gas and particles are well known (e.g., [13, 14]). A formulation of the problem for supersonic flow over a blunt body has been given in [15].

To estimate the influence of the interaction between phases in the shock layer on the thermal and erosive characteristics, we consider the ratios of the quantities  $q_s$  and  $G_{er}$ , determined from the parameters of the gas and particles at the surface of the body obtained from the solution of the problem of flow of a dusty gas stream over a blunt cone in the approximation of a two-velocity and two-temperature continuous medium, to the corresponding quantities  $q_{s\infty}$  and  $G_{er\infty}$ , determined from the parameters of the particles in the oncoming stream and the parameters of dust-free gas at the surface of the body ( $q_s$  is the thermal energy flux produced by particles incident on the surface of the body and  $G_{er}$  is the linear rate of erosive ablation).

The studies were carried out on the example of flow over a 10-degree cone with spherical blunting of radius  $R = 0.27$  m by a gas stream containing particles at  $M_\infty = 3$  and 4. The particle diameter was in the range 5-1500  $\mu\text{m}$ . The remaining parameters were as follows: mass fraction of the solid phase (ratio of the particle mass per unit volume to the gas density)  $\rho_{S\infty} = 0.4, 0.8, 1.2$ ; density of particle material and gas  $\rho_S^0 = 2 \cdot 10^3$  kg/m<sup>3</sup> and  $\rho_\infty = 0.13$  kg/m<sup>3</sup>; gas viscosity in the oncoming stream  $\mu_\infty = 1.6 \cdot 10^{-4}$  kg/(m·sec); Prandtl number  $Pr = 0.72$ ; ratio of specific heats of the particle material and the gas at constant pressure  $c_S/c_D = 1$ ;  $h_w = 0.5H_{0\infty}$  is the particle enthalpy at the wall temperature  $T_w$ , which appears in [1].

Let us first analyze the behavior of the gas and particle temperatures at the critical point of a sphere. The gas and particle temperatures, normalized to the total gas temperature  $T_0$ , at the stagnation point on a sphere for  $M_\infty = 3$  are given in Fig. 1. From that figure it follows that for particles of the small fraction (for  $d_\infty \leq 20$   $\mu\text{m}$ ), the gas and particle temperatures are close and hardly depend on the mass fraction of the solid phase in the oncoming stream. As the size of the particles increases, their temperature decreases monotonically and approaches the temperature in the oncoming stream. The gas temperature behaves nonmonotonically: with increasing particle size it first decreases somewhat, due to

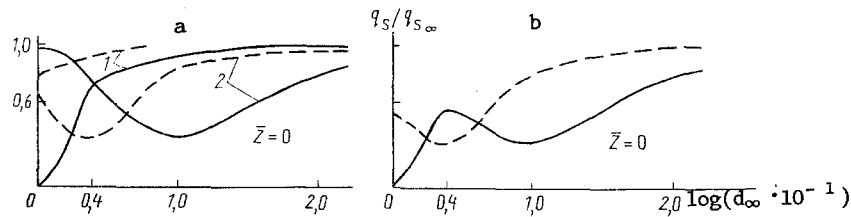


Fig. 3. Deposition coefficient and total energy imparted by particles to the surface (a) and heat flux (b) as functions of particle size at the critical point on a sphere: 1) deposition coefficient; 2) total particle energy.

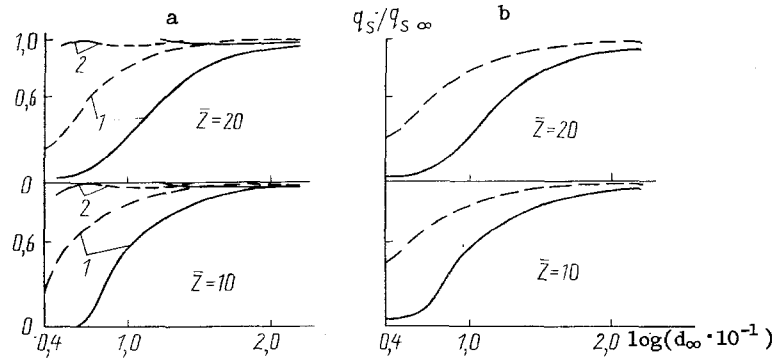


Fig. 4. Deposition coefficient and total energy imparted by particles to the lateral surface of the cone (a) and heat flux (b) as functions of particle size: 1) deposition coefficient; 2) total particle energy.

heat exchange between phases, and it then increases, reaches a maximum at  $d_\infty \approx 150 \mu\text{m}$ , and then decreases, approaching the stagnation temperature in the oncoming stream. The increase in gas temperature is associated with dissipation of the kinetic energy of the particles, to which there are two aspects: on the one hand, the kinetic energy of the particles increases with increasing size, while on the other hand, the travel time of a particle from the shock wave to the surface of the body decreases, and the number of particles decreases if the mass fraction of the solid phase is constant. Because of this, the dissipated kinetic energy first increases with increasing particle size and then decreases, resulting in the corresponding variation of both the gas temperature and the convective heat flux.

Let us return to a discussion of the thermal and erosive action of the particles. In all of the subsequent figures, the results obtained for  $\rho_{S\infty} = 0.8$  and  $M_\infty = 3$  are plotted with solid lines and those for  $M_\infty = 4$  with dashed lines.

The ratios  $q_s/q_{s\infty}$  and  $G_{er}/G_{er\infty}$  as functions of particle size are given in Figs. 2-4. One may see from the figures that the interaction between phases in the shock layer leads to a considerable decrease in the erosive (Fig. 2) and direct thermal (Figs. 3 and 4) action of the particles on the surface of the body. The influence of the interaction between phases decreases with an increase in the Mach number from 3 to 4. This is related to the decrease in the thickness of the shock layer with increasing  $M_\infty$ , which results, in turn, in a decrease in the distance a particle travels from the shock wave to the surface of the body. The influence of the interaction between phases increases with greater distance from the critical point, which can be noted by comparing the curves for  $Z = z/R = 10$  and 20. Small particles (for  $d_\infty < 100 \mu\text{m}$  at  $M_\infty = 3$  and for  $d_\infty < 40 \mu\text{m}$  at  $M_\infty = 4$  in the examples under consideration) exert almost no erosive action.

The main features of the influence of the interaction between phases on the components of the heat flux imparted by particles to the surface can be traced on the example of flow in the vicinity of the critical point of the spherical segment (Fig. 3). In Fig. 3a we show the coefficient of particle deposition  $\rho_s V_{ns} / \rho_{s\infty} V_{ns\infty}$  and the total energy  $E_s = [h_s + (V_s^2/2) - h_w] / (H_{0\infty} - h_w)$  imparted by particles to the surface as functions of the size of the incident particles (curves 1 and 2, respectively) at Mach numbers 3 and 4. One may see from the figure that the deposition coefficient decreases monotonically with decreasing particle size, with the change being more intense at  $M_\infty = 3$ . This is because the distance from the

shock wave to the body is larger in that case than at  $M_\infty = 4$  and the particles travel farther through the gas, the velocity of which in the shock layer is far lower than the particle velocity. For  $10 \mu\text{m}$  particles the deposition coefficient is close to zero for  $M_\infty = 3$ , whereas it is 0.78 for  $M_\infty = 4$ .

As for the total energy imparted by the particles to the surface of the body, it varies nonmonotonically with decreasing particle size. Mechanisms similar to those that result in the nonmonotonic dependence of the gas temperature in the shock layer on particle size may operate here. For small particles, the effect of particle heating in the gas, the temperature of which has increased in the transition through the shock wave, predominates, while for large particles, the effect of a decrease in kinetic energy due to their deceleration in the shock layer predominates. This results in a minimum in the dependence of the total energy on particle size (at  $d_\infty \approx 100 \mu\text{m}$  for  $M_\infty = 3$  and at  $d_\infty \approx 25 \mu\text{m}$  for  $M_\infty = 4$ ), the processes being more intense for  $M_\infty = 3$ .

This type of variation of the total particle energy and the deposition coefficient leads, in general, to nonmonotonic variation of the total heat flux imparted by the particles over the surface of the body (Fig. 3b), which is a product of the deposition coefficient and the total particle energy, in accordance with (1). The relationship between these quantities is such that the relative heat flux imparted by particles to the surface first decreases monotonically with decreasing particle size, passes through a minimum (at  $d_\infty = 100 \mu\text{m}$  for  $M_\infty = 3$  and at  $d_\infty = 25 \mu\text{m}$  for  $M_\infty = 4$ ), increases to a maximum, and then decreases to zero as the deposition coefficient approaches zero. In the transition to the lateral surface of the cone, the heat flux imparted by the particles has a monotonic dependence on particle size. This is because the variation of the kinetic energy of the gas and particles and their heating are less intense on the lateral surface than in the vicinity of the blunting. The total particle energy varies slowly under these conditions (Fig. 4a, curve 2) and the dependence of the deposition coefficient on particle size (Fig. 4a, curve 1) becomes decisive in the variation of  $q_s/q_{s\infty}$ .

#### NOTATION

$c_s, c_p$ , specific heats of particles and gas;  $\rho_s, V_s, T_s, h_s$ , density, velocity, temperature, and enthalpy of particles;  $T_w, h_w$ , wall temperature and particle enthalpy at the wall temperature;  $H_{er}$ , effective enthalpy of erosive ablation;  $n_w$ , normal to the surface of the body;  $M_\infty$ , Mach number;  $R$ , radius of spherical blunting;  $d_\infty$ , particle diameter;  $Z = Z/R$ , axial coordinate of the body;  $H_{0\infty}$ , total stagnation enthalpy of the gas.

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NUMERICAL SIMULATION OF THE FLOW OF A SUPERSONIC  
STREAM OF VISCOUS COMPRESSIBLE GAS OVER A CAVITY

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UDC 517.9:519.95

Numerical simulation of the flow of a supersonic stream of viscous, compressible, heat-conducting gas over a cavity is carried out on the basis of kinetically consistent difference schemes. Different types of flow ("open" and "closed" cavities) are considered, the heat fluxes to the walls of the recess are determined, and a nonsteady regime of flow over a cavity is simulated. The results obtained are compared with known experimental relationships.

### 1. Introduction

Experimentally observed detached flows in recesses, over which a supersonic stream of viscous gas flows, have both a steady and a nonsteady character [1-5]. The types of such flows have been classified in [5]. Numerical simulation of these flows is urgently needed for practical applications of problems of computational hydrodynamics, the complexity of which is due to the need for simultaneous allowance for the region of inviscid flow and for viscous effects within the boundary layer. Let us cite several of the most typical examples of the calculation of such flows. Conjugation of solutions for the compressible boundary layer above the recess, for the compressible Navier-Stokes equation within the recess, and for inviscid flow above the recess outside of the boundary layer has been used in [6] to calculate steady detached flow in a cavity. In such an approach, however, an iterative process cannot be constructed to join the solutions for calculating nonsteady flows, and in that case it is natural to use the complete Navier-Stokes equations for numerical simulation. In [3], in particular, the averaged Navier-Stokes equations were used in combination with McCormack's method, and in [7] the method of splitting with respect to physical processes and spatial variables was used. To calculate pulsation flows in the model of an ideal gas, which seems simpler from the standpoint of practical computation, periodic variations of the parameters at the entrance boundary must be specified to excite oscillations in the cavity [8].

In the present paper, which is a continuation of [9], we carry out a series of calculations of supersonic flow over cavities by a stream of viscous compressible gas on the basis of kinetically consistent difference schemes (KCDSs) with corrections [10]. Although the flow in a cavity has a three-dimensional character, as noted experimentally, the two-dimensional formulation of the problem has been considered to clarify the main features of that flow and reduce the volume of calculations. Laminar flow regimes have been investigated. In the calculations we varied the geometrical dimensions of the recesses and studied flow in cavities of two different types: "open" and "closed." The heat fluxes to the cavity walls were determined in the case of isothermal boundary conditions. The pulsation regime of flow was simulated for an adiabatic cavity of the open type. In this case, in contrast to [8], we did not have to specify perturbations of the parameters at the entrance boundary to excite the oscillations. The results of the calculations have been compared with the known experimental data [1, 5]. In the case of pulsation flow, the results were compared with known data of natural experiments [2, 4] and with analytic functions [3].

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M. V. Keldysh Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 61, No. 4, pp. 570-577, October, 1991. Original article submitted September 18, 1990.